## Activity 32 Combining transformations

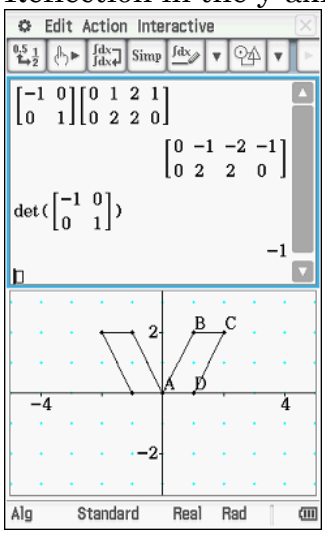
1. $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 2 & 0 \end{bmatrix}$

2.

a) $\det T_1 = -1$

$T_1P = \begin{bmatrix} 0 & -1 & -2 & -1 \\ 0 & 2 & 2 & 0 \end{bmatrix}$

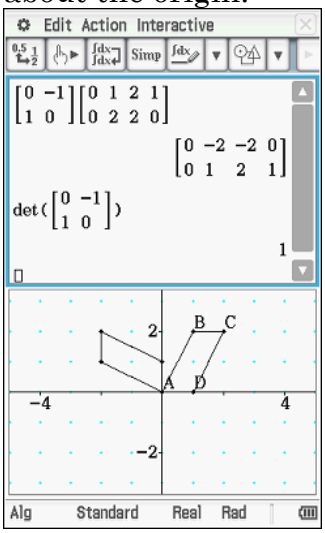
Reflection in the y-axis.




b) $\det T_2 = 1$

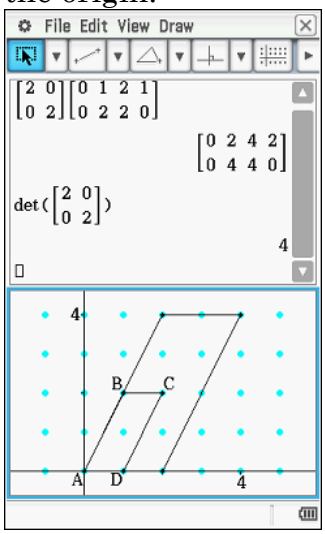
$T_2P = \begin{bmatrix} 0 & -2 & -2 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

Rotation of 90° anticlockwise about the origin.



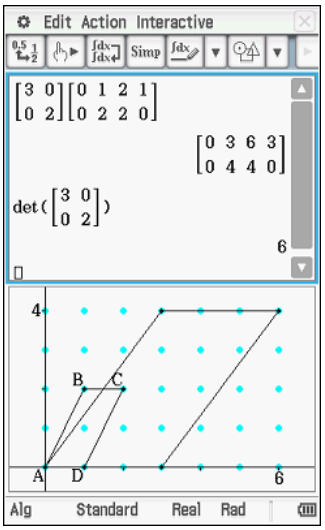

c) $\det T_3 = 4$ $\quad T_3P = \begin{bmatrix} 0 & 2 & 4 & 2 \\ 0 & 4 & 4 & 0 \end{bmatrix}$

Enlargement: scale factor 2, centre the origin.



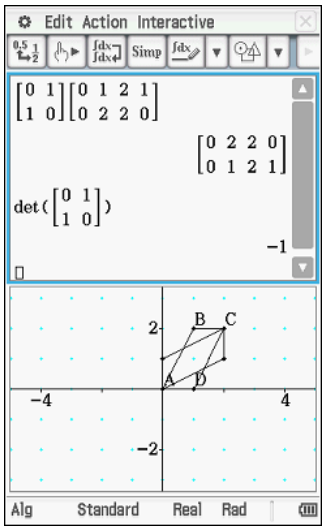

d) $\det T_4 = 1$ $\quad T_4P = \begin{bmatrix} 0 & 3 & 6 & 3 \\ 0 & 4 & 4 & 0 \end{bmatrix}$

Horizontal dilation factor 3 and vertical dilation factor 2.



e) $\det T_5 = -1 \quad T_5P = \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

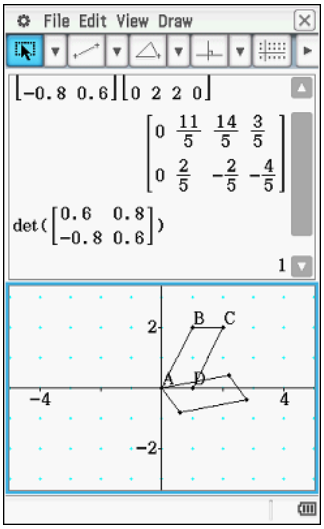
Reflection in the line $y = x$




f) $\det T_6 = 1$

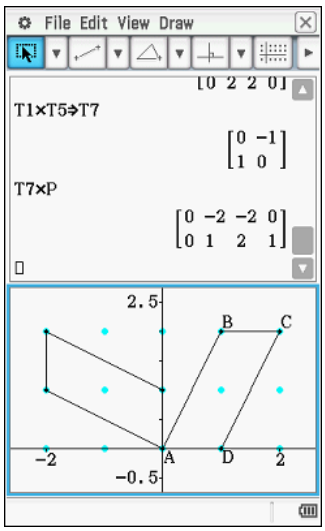
$T_6P = \begin{bmatrix} 0 & 2.2 & 2.8 & 0.6 \\ 0 & 0.4 & -0.4 & -0.8 \end{bmatrix}$

Rotation clockwise about the origin of $53^\circ = \tan^{-1}\left(\frac{4}{3}\right)$



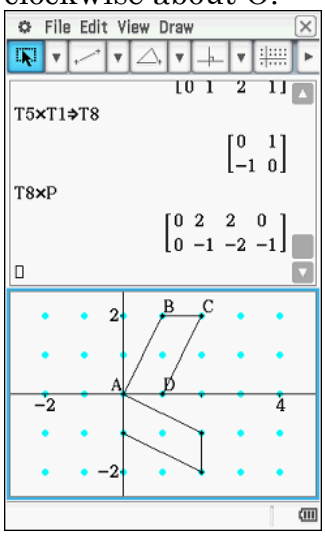

3.

a) $T_1T_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Rotation of 90° anticlockwise about O.




b) $T_5T_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Rotation of 90° clockwise about O.




c) $T_7$ and $T_8$ are both rotations of 90° but in opposite directions. They are not the same transformation. The order in which multiple transformations are performed can make a difference to the result. Combining linear transformations is not commutative.

d) $\mathbf{T}_9 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the identity matrix or identity transformation. Four rotations of 90° is a complete revolution and so the object will return to the starting position.

4.

a) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

b) $\begin{bmatrix} -0.5 & 0.866 \\ -0.866 & -0.5 \end{bmatrix}$ or $\begin{bmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$

c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

e) $\begin{bmatrix} -0.5 & 0.866 \\ 0.866 & 0.5 \end{bmatrix}$ or $\begin{bmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

5.

| $1 \to 2$ | $2 \to 3$ | $3 \to 4$ | $4 \to 5$ |
|---|---|---|---|
| $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ | $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$ |
| Reflection in $y$-axis | Clockwise rotation of 90° | E.g. clockwise rotation of 90° then reflect in the $y$-axis | Reflect in the $x$-axis then rotate 45° clockwise. |